

# Inter-pattern speculation

*beyond Minority, Majority and \$-games*

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- Market market mechanism
- Minority Games
- Pattern Games
- Back to Minority Games ?

Econophysics: [www.unifr.ch/econophysics](http://www.unifr.ch/econophysics)

Eco-physics

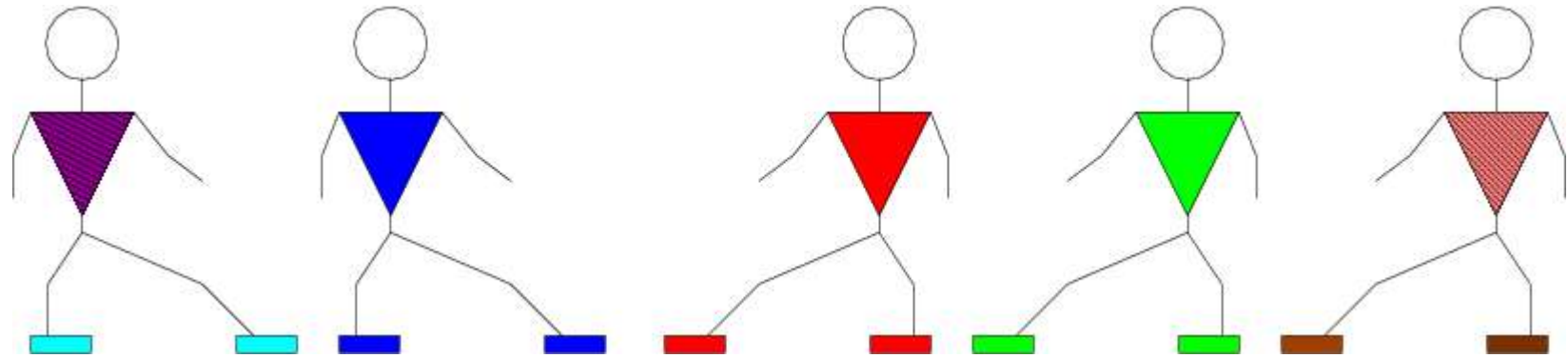
Eco-nomo-physics

Condensed-matter physicists: cooperative phenomena, phase transitions, non-equilibrium (driven) systems

1) high frequency data analysis

2) agent-based modelling: +1/-1 (spin) models, minority/majority/\$-games  
=> powerful analytical tools for heterogeneous systems

# Minority Game



- N-player game
- two choices
- Aim: to be in the minority
- Outcome  $A = \#UP - \#DOWN = \#\text{⬆} - \#\text{⬇}$   
⇒ Generic model of competition

# MG: traders

- Agent  $i =$

strategy 1 + score  $U_{i,1}$

strategy 2 + score  $U_{i,2}$

...

...

- **Induction**: use the strategy with the **highest** score
- at time  $t$ , action  $a_i(t)$
- payoff  $-a_i(t)A(t)$        $A = \sum_j a_j$

Strategy = reaction to market states, patterns,  
signals

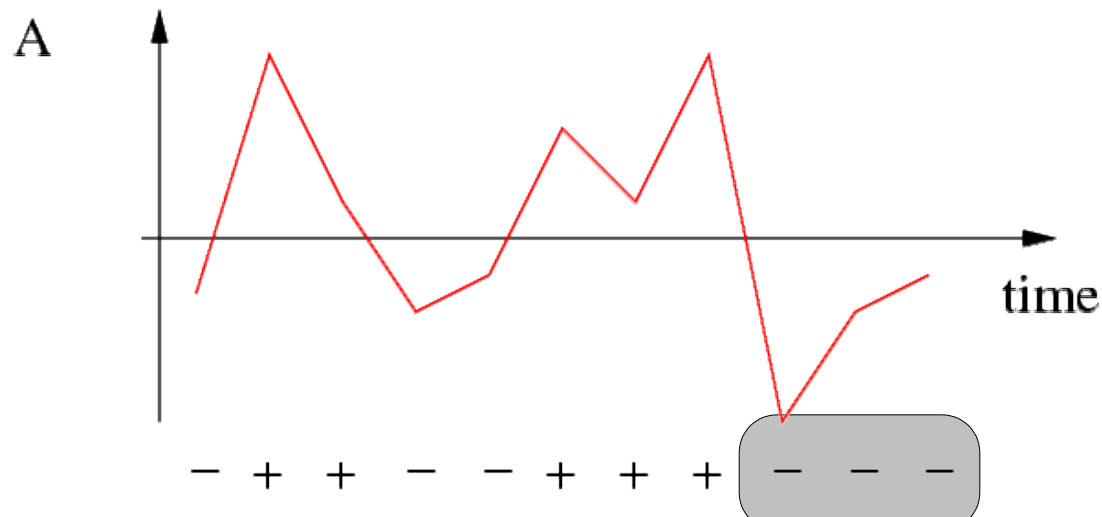
= look-up table

At time  $t$ , pattern  $\mu(\mathbf{t}) \in \{1, \dots, P\}$

State 1 2 3 4 5 6 7 8

Action + - - - + + - +

example:  $M$ -step price change encoding,  $P=2^M$

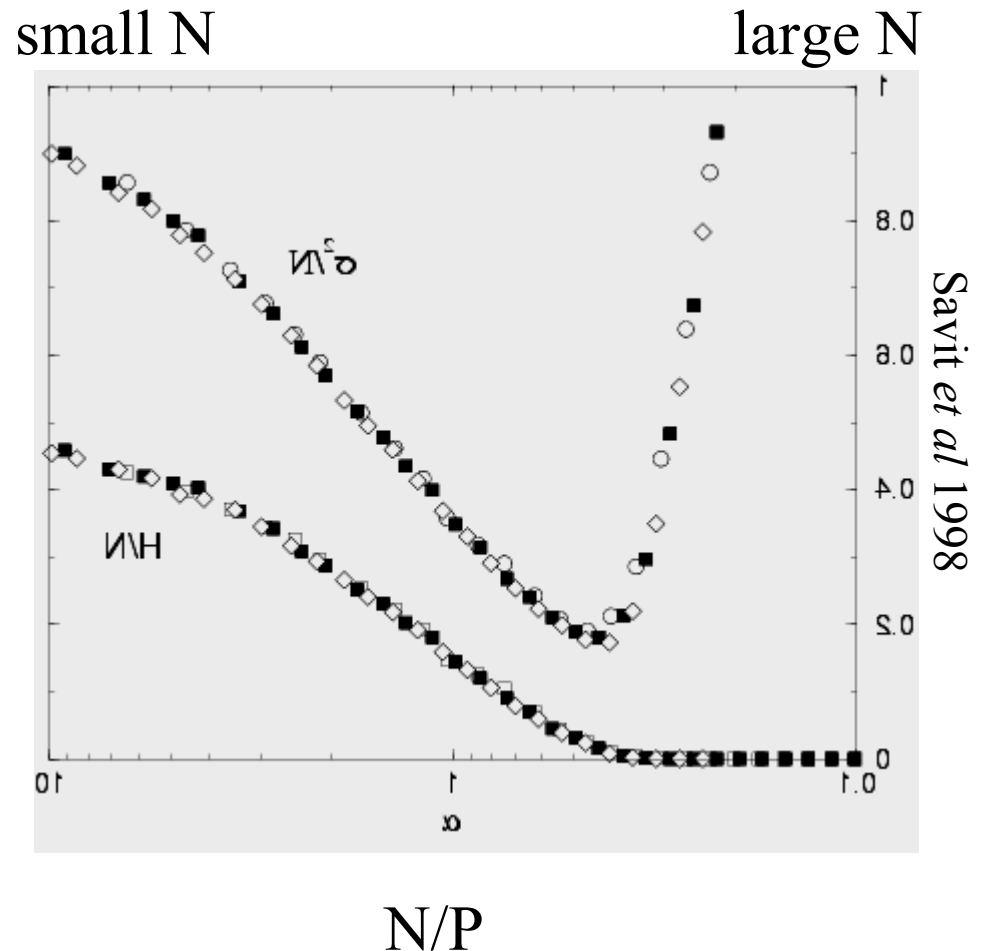


# Properties of the Minority Game

$\sigma^2 = \langle A^2 \rangle$  volatility  
 = N random  
 $< N$  better, cooperation  
 $> N$  worse

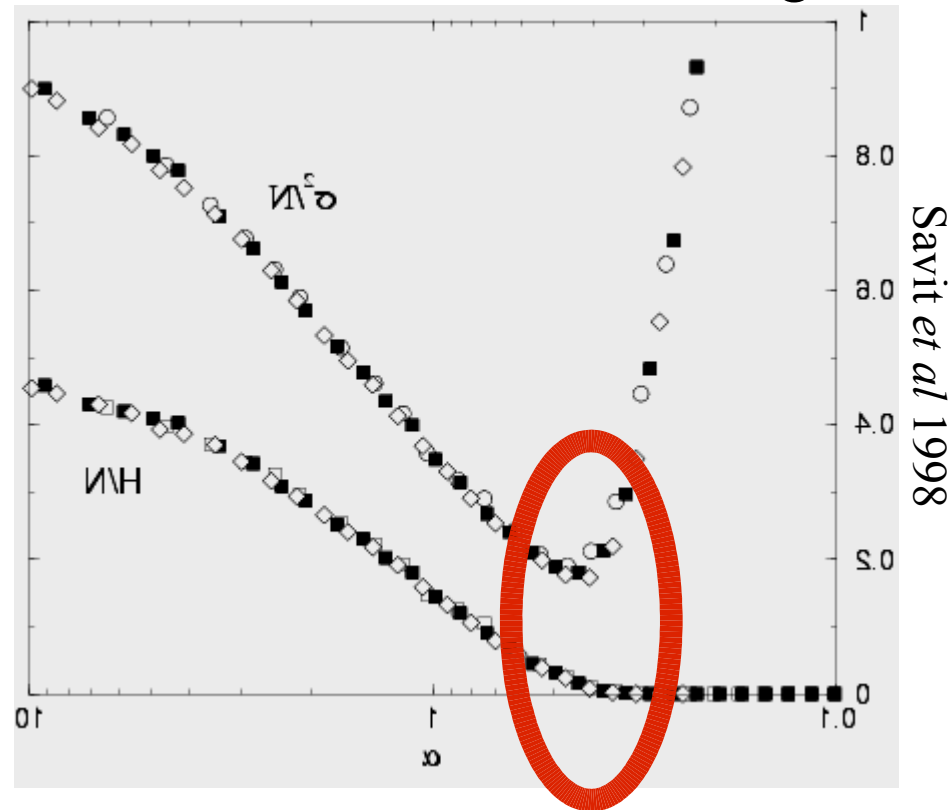
$H = \sum_{\text{pattern}} \langle A | \text{pattern} \rangle^2$   
 predictability  
 measure of asymmetry

$H > 0$ : market predictable



small N

large N



Special point:

Finance: transition between efficient/inefficient markets

Physics: phase transition

at this phase transition, correlation functions etc: power-laws  
exactly solvable

exact methods for any systems with heterogeneous agents

# Markets as ecology of information

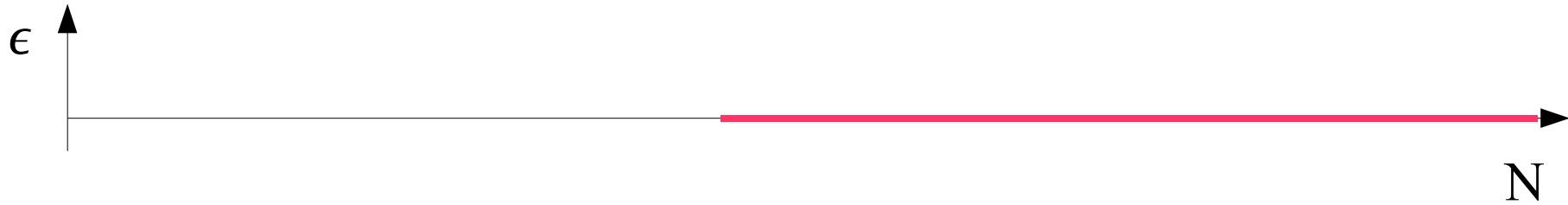
- The traders minimize predictability
- Predictability attracts new traders
- Predictability decreases as #traders increases
- Drives the system -> efficiency
- Self-organized efficiency ?

# Simplest market-like minority game

- Allow speculators not to play  
*(Slanina and Zhang 1999, Johnson et al 1999)*
- Producers always IN:  $\Rightarrow$  predictability
- Speculators enter and withdraw from the market
  - Criterion: **IN** if strategy avg return  $> \varepsilon$   
**OUT** if strategy avg return  $< \varepsilon$
- Hope: self-organization around the transition point

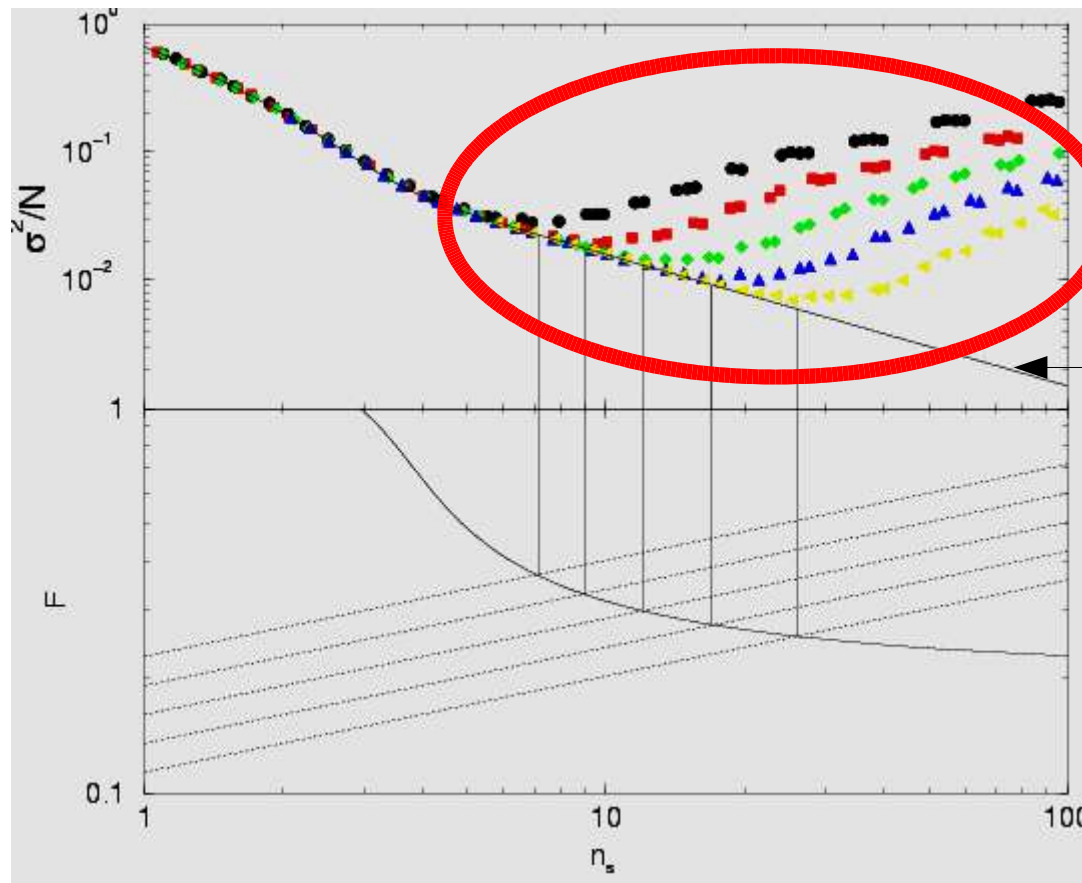
# Properties of grand canonical MGs

1 semi-line of transition instead of a single point



Small N

Large N



**Non-Gaussian returns**

P, 2P, 4P, 8P, 16P

Exact solution (Gaussian)

Signal to noise ratio  
=  
exploitable predictability

# Properties of grand canonical MGs

critical line  $\Leftrightarrow$  market-like behaviour:

- Links power-laws in markets to phase transition  
(*cf Cont-Bouchaud 1997*)
- Phase transitions in Nature are characterized by power-laws.
- States the nature of the two phases:  
market are at the border of efficiency  
(*Marsili&Giada 2002, Bouchaud et al. 2004*)
- Plausible scenario



# Market mechanism:

- A trader opens a position  $a = +1 / -1$  (Buy/Sell):
  - $p(t)$  known
  - $p(t+1) = p(t) + f(A(t)) \cong p(t) + (\#Buy - \#Sell) / \text{liquidity}$
- The trader closes his position
  - $p(t')$  known
  - $p(t'+1) = p(t') + f(A(t'))$

- Payoff

Price impact  
Minority game

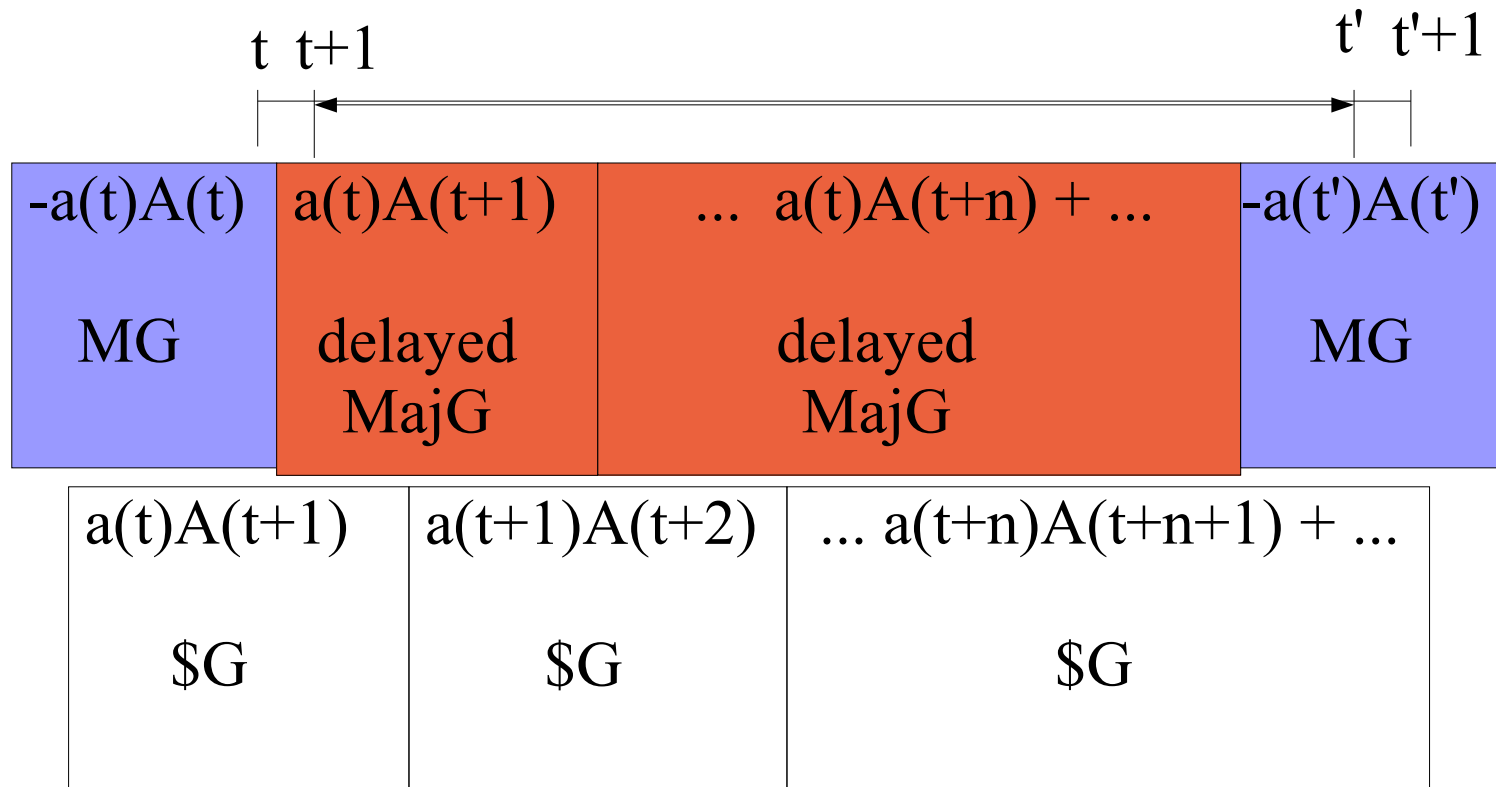
Naive capital gain  
Delayed majority game

$$a [ p(t'+1) - p(t+1) ] \cong - (-a) A(t') - a A(t) + a [ p(t') - p(t) ]$$

Price impact  
Minority game

Naive capital gain  
Delayed majority game

$$a [ p(t'+1) - p(t+1) ] \cong - (-a) A(t') - a A(t) + a [ p(t') - p(t) ]$$



# MG and financial markets

minority mechanisms:

if agents trade at each time step, they play a MG

market makers play a MG

MG: mean reverting process

describe price impact

more important for frequent traders

# How to go beyond Minority/majority/\$-games?

Main problem: strategy space.

**SIMPLE** inter-temporal strategies ?

How to open a position?

wait for a pattern/signal (crossing of EMAs, head-and-shoulders, etc)

How to close a position?

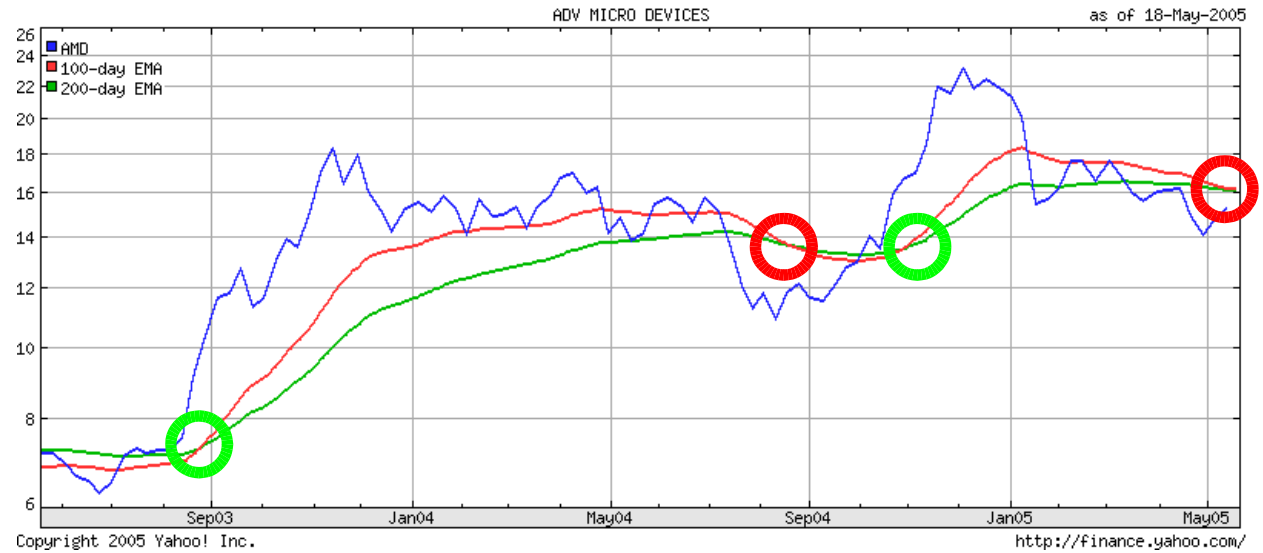
fixed time horizon, stop loss, stop gain

wait for a pattern/signal

# Examples: Exponentially Moving Averages

2 patterns

Strategy:  
compute price returns  
between the patterns



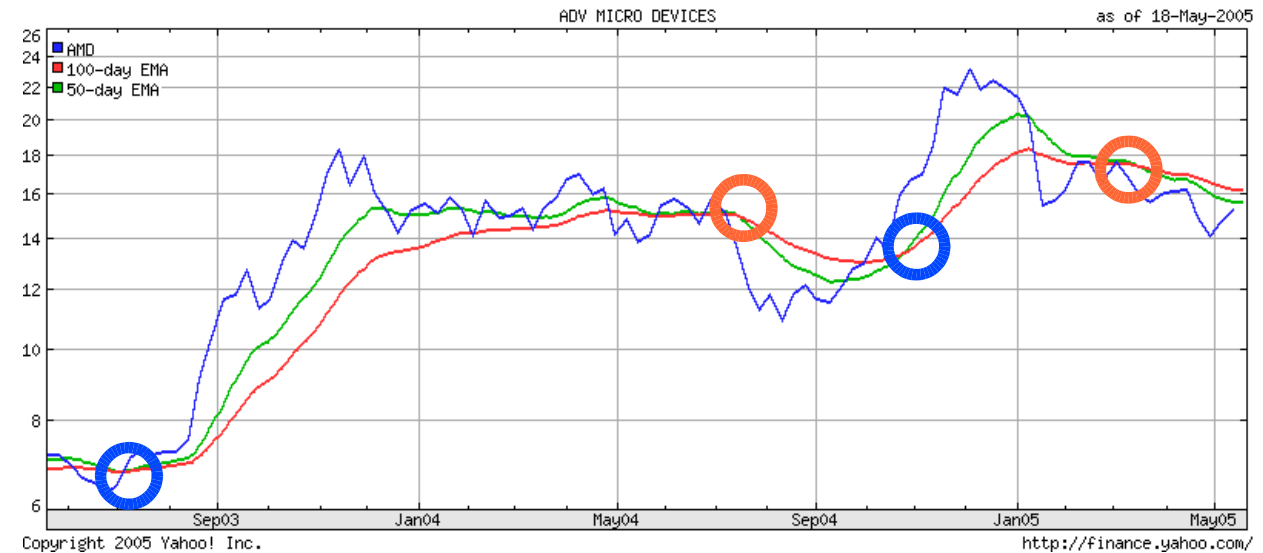
Other parameters

$\Leftrightarrow$

Other patterns

$\Rightarrow$

Not active simultaneously



# Pattern-based speculation:

market state, pattern  $\mu(\mathbf{t}) \in \{1, \dots, P\} = \text{blue circle} \text{ green circle} \text{ red circle} \dots \text{orange circle}$

Each agent

- recognizes 2 patterns  $\mu_1$  and  $\mu_2$
- measures the average return  $\langle r \mid \mu_i \rightarrow \mu_j \rangle$  between the patterns



$$\mu(t_1) = \mu_1$$

$$\mu(t_2) = \mu_2$$

- agent active when he recognises a pattern [ $\mu(t) = \mu_1$  or  $\mu(t) = \mu_2$ ]
- opens position only if  $|\langle r \mid \mu_1 \rightarrow \mu_2 \rangle| > \varepsilon$
- buy if  $\langle r \mid \mu_1 \rightarrow \mu_2 \rangle > 0$  and reversly

## Dynamics of market states:

$\mu(t)$  drawn uniformly at random from  
 $1, \dots, P$

## How to measure price returns ?

1) naive: measures  $p(t_2) - p(t_1)$

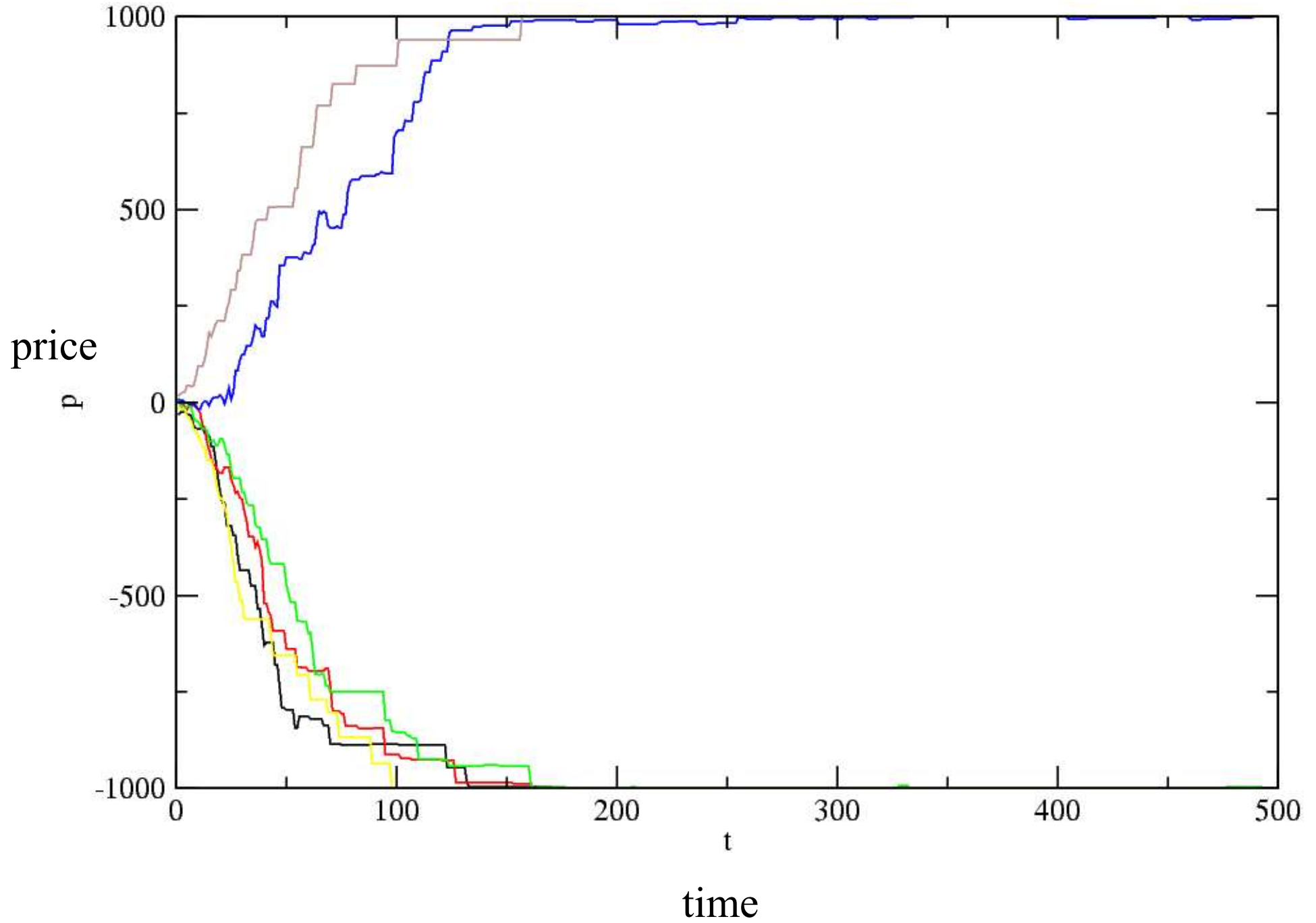
2) sophisticated: measures  $p(t_2+1) - p(t_1+1)$  : market impact

being outside the market and inside is different:

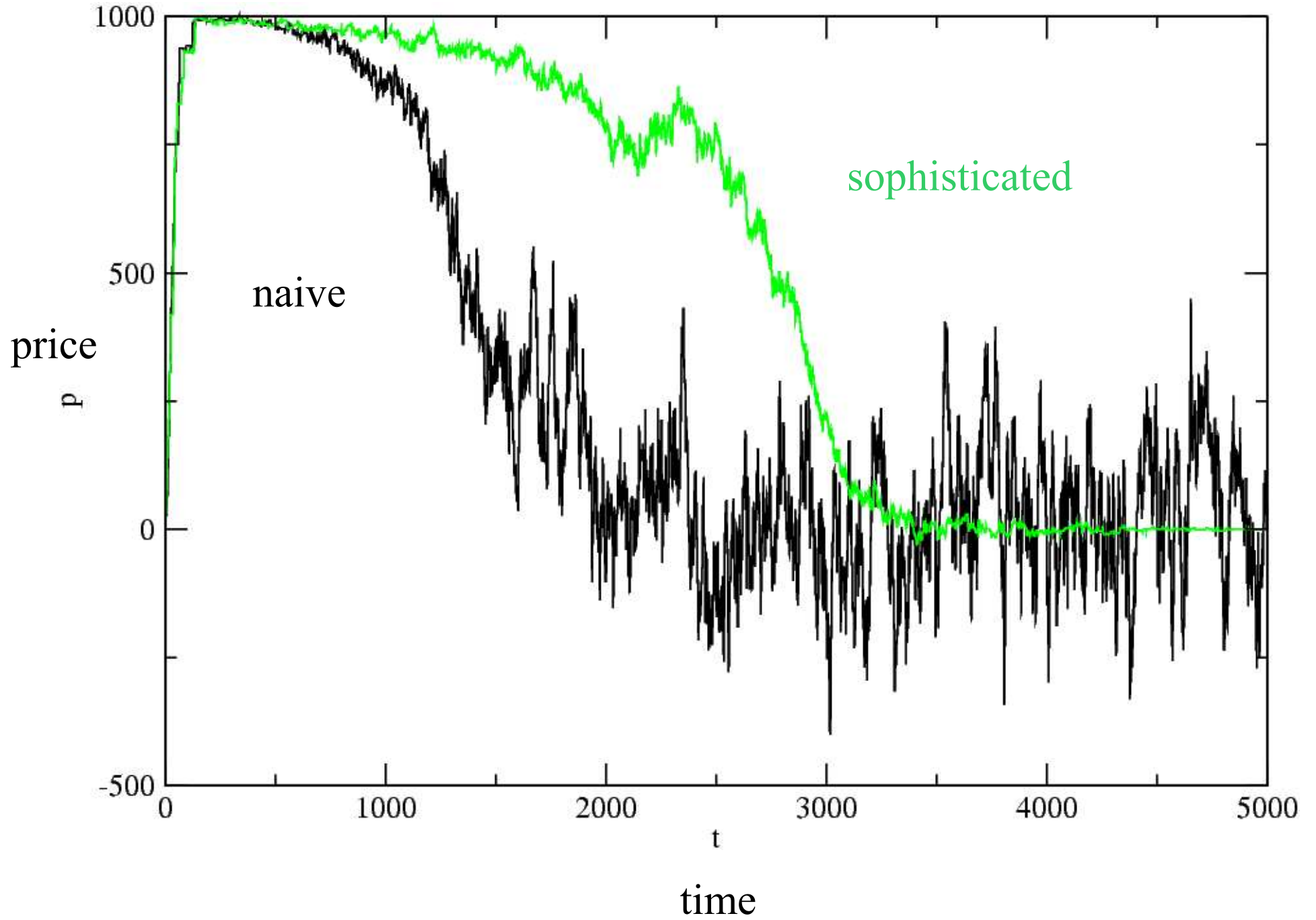
## MARKET IMPACT

=> a NAIVE agent is SOPHISTICATED in the market (feels the pain).

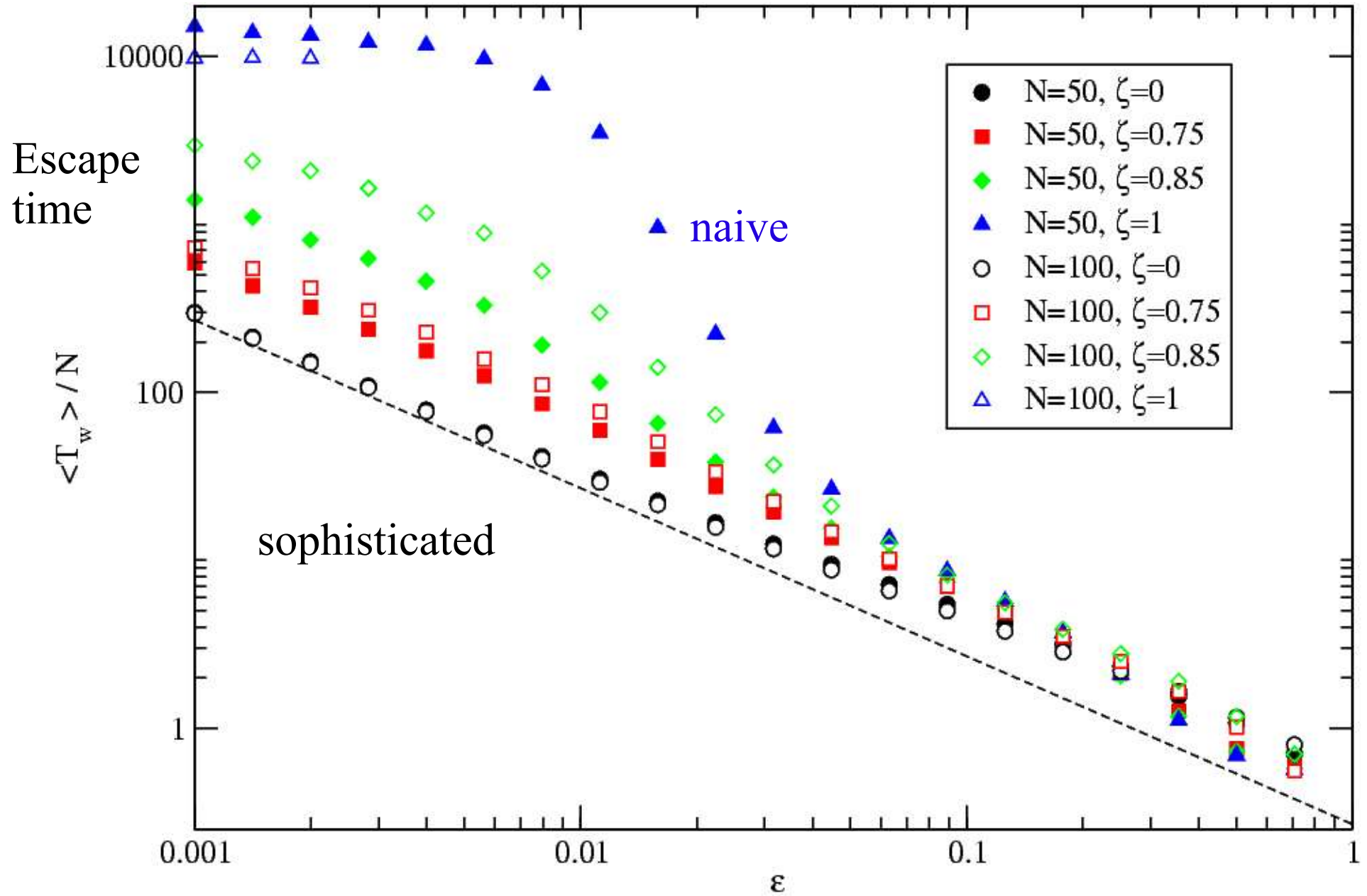
**Results:**  $\varepsilon = 0$



**Results:**  $\varepsilon > 0$



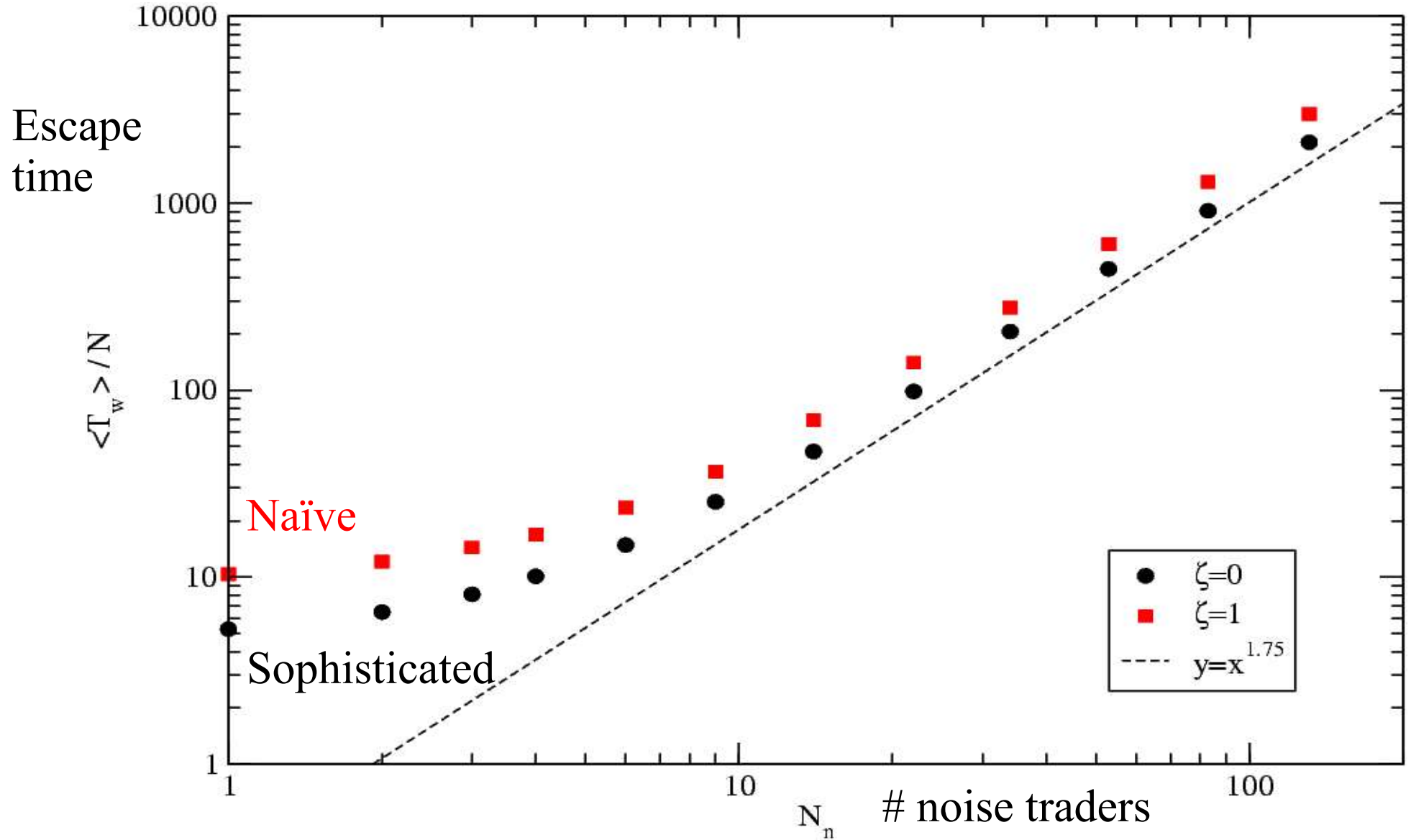
**Results:**  $\varepsilon > 0$



Problem:

How to keep the agents in the market?

Noise ?



## **Problem:**

How to make speculators *trade* ?

Inject predictability:

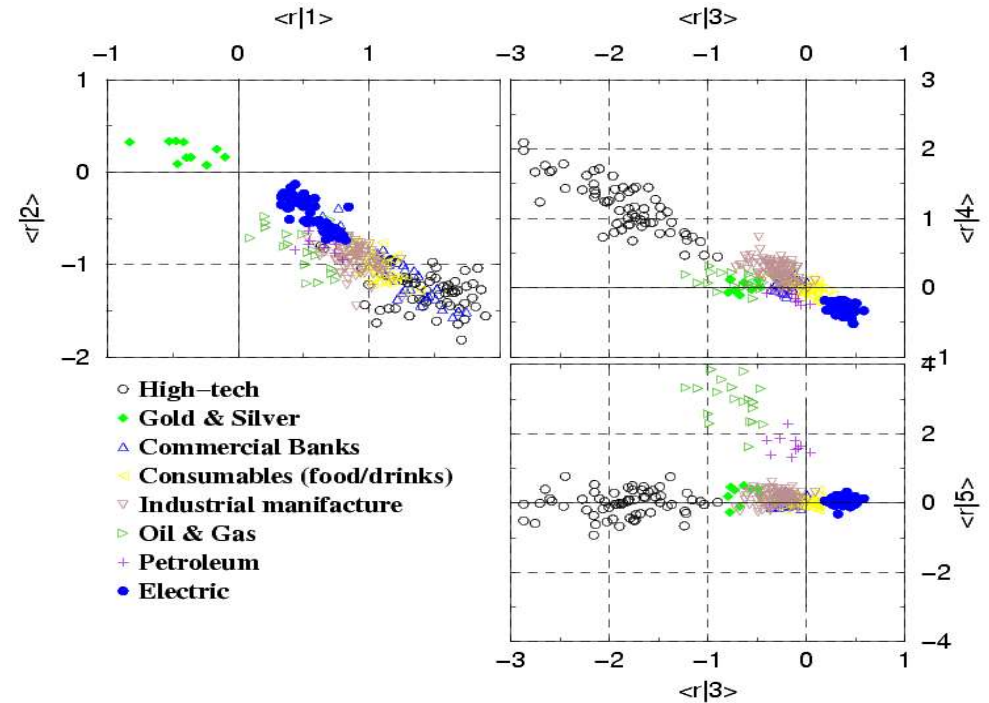
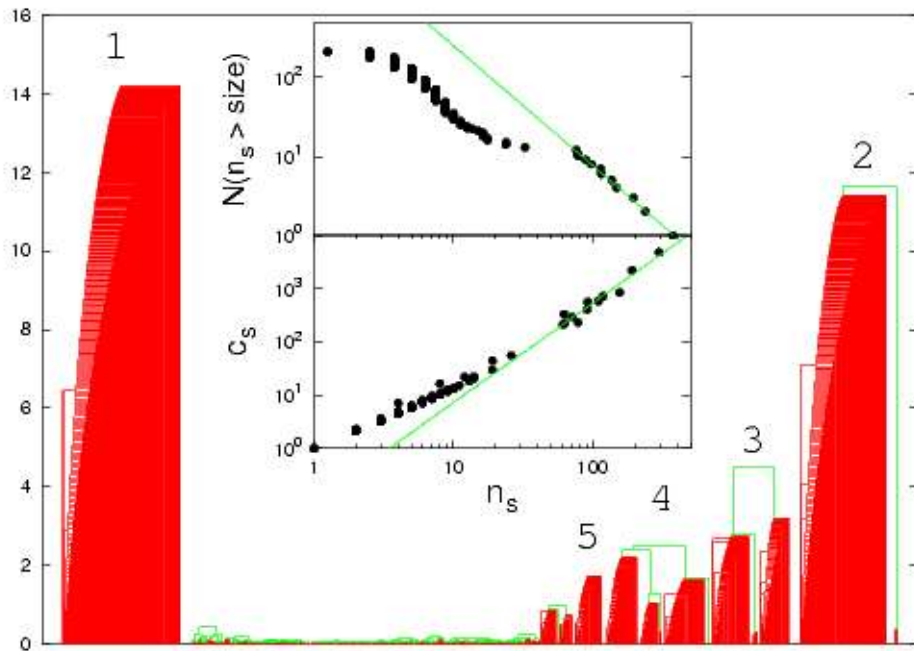
*Producers* are predictable  $\Rightarrow A(t) = A_{\text{speculators}}(t) + A_{\text{producers}}[\mu(t)]$

*Not sufficient: speculators need correlations*

## **Solution:**

Time-evolution of  $\mu$ : not uniform  $\Rightarrow W(\mu \rightarrow \nu) \neq 1/P$

# Marsili, Quantitative Finance 2001: clustering of market days



5 types of recognizable days

1 category of random days

Sequence of states has a strong structure

after 1987 and 1992 crashes, 21X661

two other occurrences, 27/04/1990 and 25/05/1990

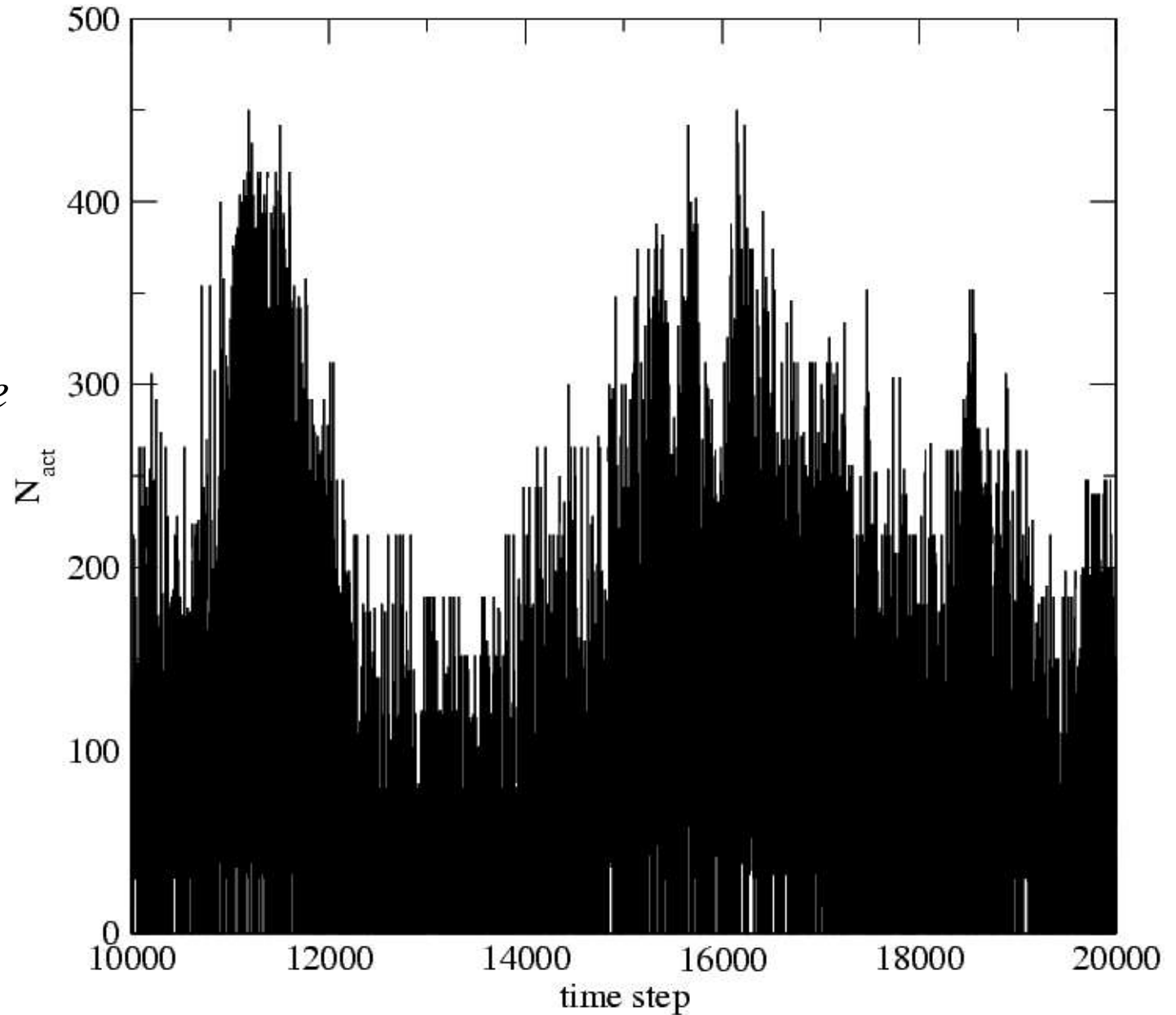
State	Date
1	900102
6	900103
4	900104
2	900105
6	900108
2	900109
6	900110
6	900111
2	900112
2	900115
1	900116
2	900117
6	900118
6	900119
2	900122
6	900123
4	900124
2	900125

# Market phenomenology?

Volume clustering:

Bouchaud *et al.* 2002

*Due to switching  
between active/inactive  
conditional on  
random walk*



When the speculators are active

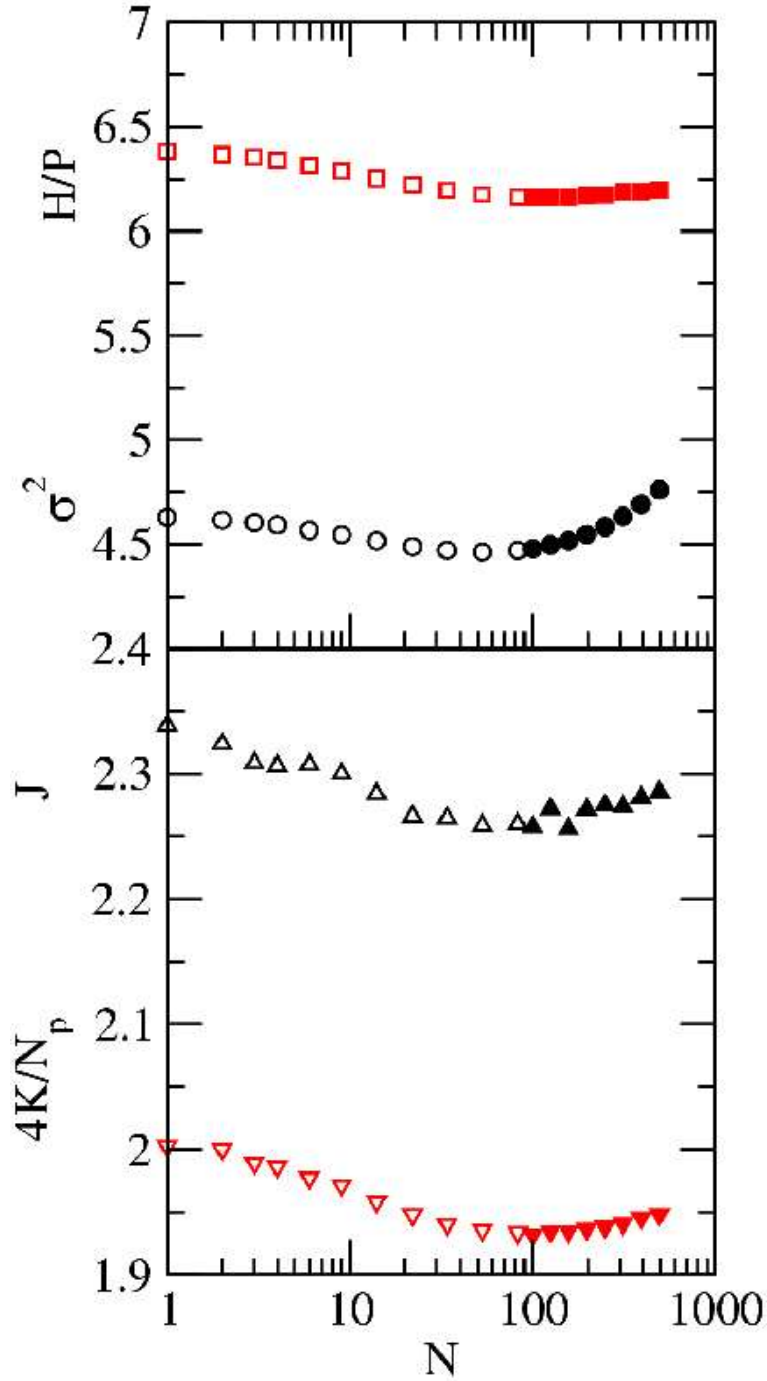
## Relevant quantities:

volatility  $\sigma^2 = \langle A^2 \rangle$

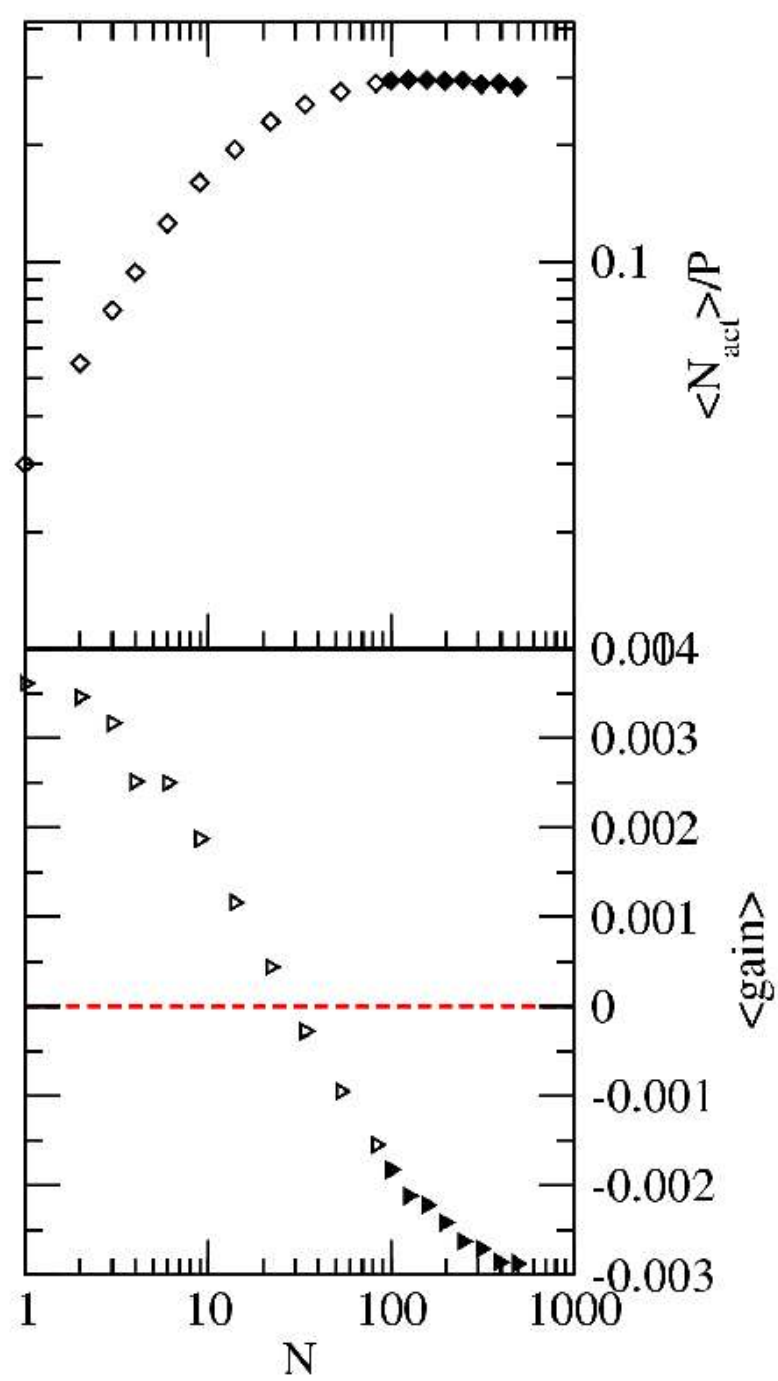
price impact  $H = \sum_{\mu} \langle A | \mu \rangle^2$

naive predictability  $K = \sum_{\mu_1} \sum_{\mu_2} \langle p(t_2) - p(t_1) | \mu_1 \rightarrow \mu_2 \rangle^2$

real predictability  $J = \sum_{\mu_1} \sum_{\mu_2} \langle p(t_2+1) - p(t_1+1) | \mu_1 \rightarrow \mu_2 \rangle^2$



# speculators



# speculators

Ecology of information similar to MG's

Hence MG is still relevant to financial markets

Why?

*Predictability*

MG = competition for limited resources

limited resources = predictability

in practice: MG when resource must not be over-exploited

New model of speculation:

few parameters

Speculation AND predictability ecology

Many extensions of the MG literature can be applied directly:

- Refrain from trading
- producers-speculators
- darwinism
- re-investment

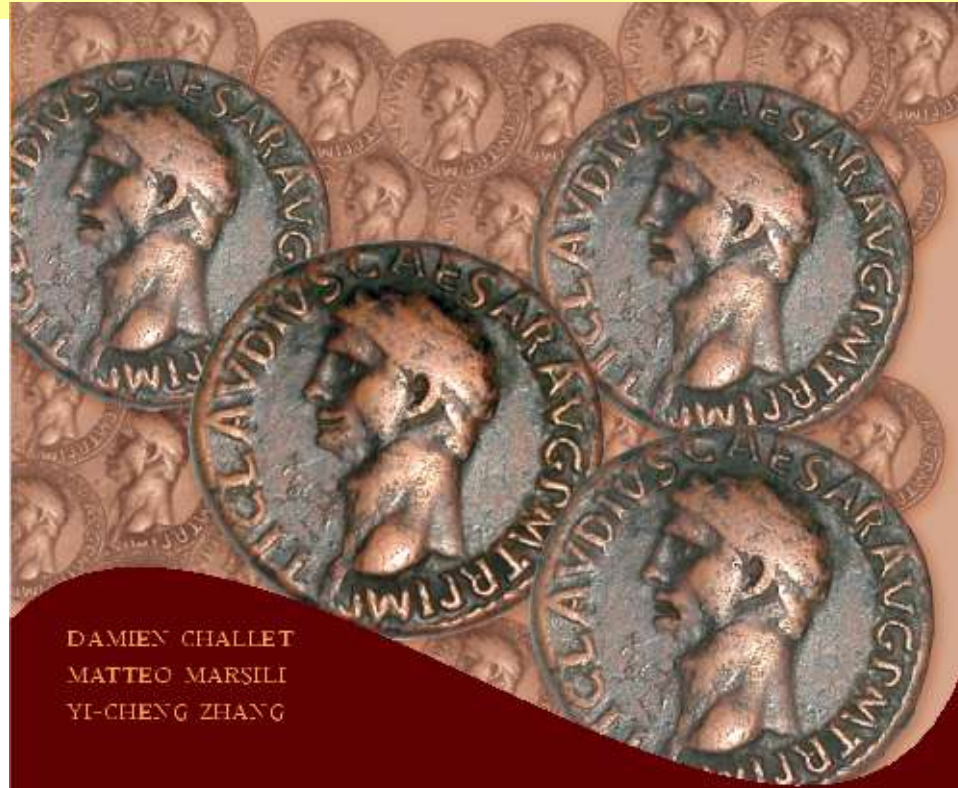
Exactly solvable

## Conclusions:

- Econophysics: powerful analytical methods for solving exactly models of adaptive heterogeneous agents
- Minority Game: competition for limited resource, look-up tables,
- Pattern-based speculation: ~ same information ecology as MG
- Minority Game: limited resource = predictability

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[www.unifr.ch/econophysics/minority](http://www.unifr.ch/econophysics/minority)



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# Minority Games

*interacting agents in financial markets*

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